AP Physics 1 Review Sessions

AP Physics 1 Test

Section 1 - MCQs:
- 50 multiple choice questions
- 1 hour 30 minutes
- 50% of test score

Questions are either discrete questions or question sets, in which students are provided with a stimulus or a set of data and a series of related questions.

Multi-select questions: 5 of the 50 questions have two correct answers - must choose both to get it right

Section 2 - FRQs:
- 5 free response questions
- 1 hour 30 minutes
- 50% of test score

Question Types:
- Experimental Design (1)
- Qualitative/Quantitative Translation (1)
- Short Answer: Paragraph Argument (1)
- Short Answer (2)

Suggested timing will be given

AP Physics 1 Exam Tips

From: https://apstudents.collegeboard.org/courses/ap-physics-1-algebra-based/exam-tips

- Plan your time for FRQs - choose an order based on difficulty and monitor your time
- Write down all your work - you can get partial credit; cross out mistakes
- Try each part of a multi-part question; if the answer for (b) depends on (a), use whatever you got for (a) even if you think it might be wrong
- Include units whenever possible
- Pay attention to the verbs in FRQs - they have specific meanings and ask you to do specific tasks (see link)

Motion Definitions

Frame of Reference: A choice of coordinate axes that defines the starting point for measuring a quantity (Video on Classroom)

Position (x)**:** A point in space; often refers to where an object is/was

Distance (d)**:** The path length traveled between two positions; how far; distance is a scalar quantity

Displacement (Δx): Change in position; $\mathbf{\Delta x} = x_f - x_i$; displacement is a vector

Vector/Scalar: Scalars have size (magnitude); Vectors have both magnitude and direction. Scalar variables are represented by plain text; vector variables are **bold**

Motion Definitions

Average speed ($\bar{v}$): The length of the path an object travels divided by the total elapsed time; v = d/t

Average velocity ($\bar{\mathbf{v}}$): The displacement during a time interval divided by the length of that time interval; $\bar{\mathbf{v}} = \mathbf{\Delta x}/\Delta t = (x_f - x_i)/(t_f - t_i)$; speed and direction

Instantaneous velocity(v): The speed and direction of an object at a particular instant in time

$$\boldsymbol{v} = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}$$

Acceleration

Acceleration (a): The changing of an object's velocity over time

Average acceleration ($\bar{a}$): The change in velocity during a time interval divided by the length of that time interval; $\bar{\mathbf{a}} = \mathbf{\Delta v}/\Delta t = (\mathbf{v}_f - \mathbf{v}_i)/(t_f - t_i)$

Instantaneous acceleration (a): The rate of change of velocity of an object at a particular instant in time

$$\boldsymbol{a} = \lim_{\Delta t \to 0} \frac{\Delta \boldsymbol{v}}{\Delta t}$$

Freely Falling Objects

A **freely falling object** is any object moving freely under the influence of gravity alone, regardless of its initial motion

For example:

- Ball dropped
- Ball thrown into the air (after it's released)

"Free fall" only happens in cases where air resistance is negligible (nearly zero).

In these cases, the **free-fall acceleration** of the object (represented as *g*) is approximately 9.80 m/s^2, at least near Earth's surface.

Kinematics for free fall

If an object is freely falling due to gravity, we know its acceleration will be $\mathbf{a} = -g = -9.80\ m/s^2$

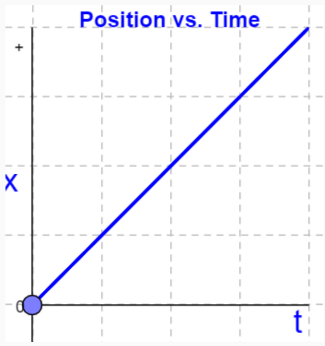
We can use that value in our equations of motion in place of acceleration:

- $v = v_0 + at$
- $x = x_0 + v_0 t + ½at^2$

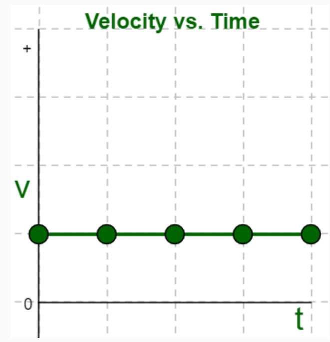
This should also allow us to represent the motion of free fall with graphs and with motion diagrams, or to solve equations for objects in free fall.

Transitioning Between Graphs

- If we have a graph of position vs. time, how can we find velocity?



$$v = \frac{\Delta x}{\Delta t}$$



- Slope of a position vs. time graph tells us velocity

Transitioning Between Graphs

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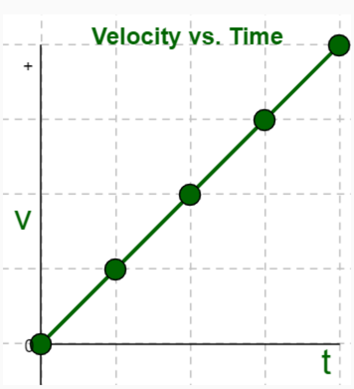

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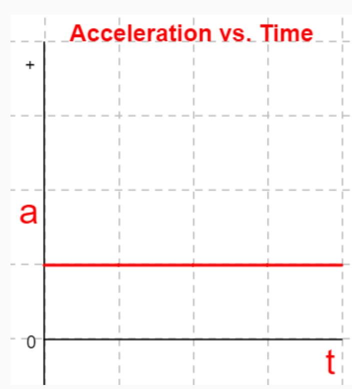

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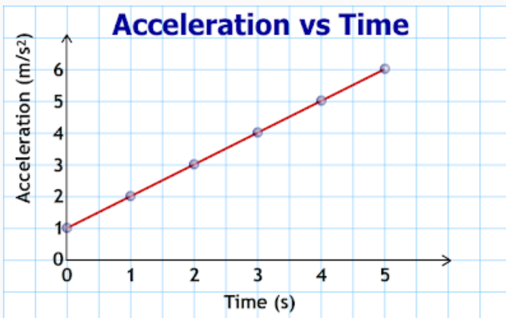
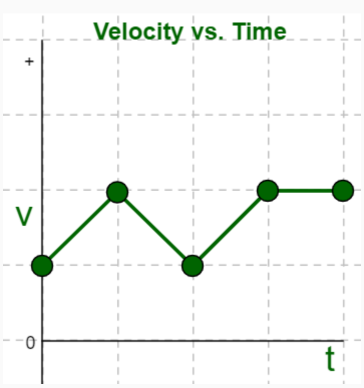


$$a = \frac{\Delta v}{\Delta t}$$

- Area under an acceleration vs time graph tells us change in velocity

Transitioning Between Graphs

- If we have a graph of velocity vs. time, how can we find position?



$$v = \frac{\Delta x}{\Delta t}$$

- Area under a velocity vs. time graph tells us change in position.

Projectile Motion

Vertical Components:

$v_y = v_{0y} + a_y t$

$\Delta y = v_{0y}t + .5a_y t^2$

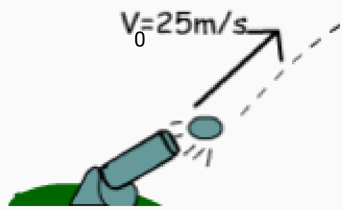
$v_y^2 = v_{0y}^2 + 2a_y \Delta y$

Horizontal Components ($a_x = 0$):

$v_x = v_{0x}$ (constant)

$\Delta x = v_{0x}t$

How do we find v_{0y} and v_{0x}?



$v_{0x} = v_0\cos(\theta)$

$v_{0y} = v_0\sin(\theta)$

So now we can write:

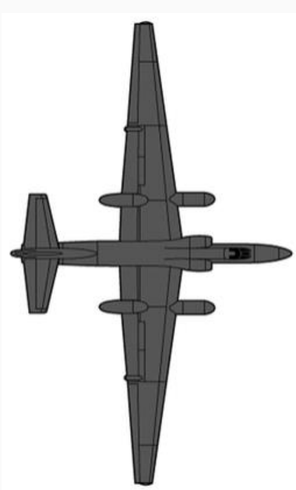
$v_y = v_0\sin(\theta) + a_y t$

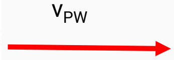
$\Delta y = v_0\sin(\theta)t + .5a_y t^2$

$\Delta x = v_0\cos(\theta)t$

Relative Velocity (Side wind)

Plane Velocity





v_{PW}

125 km/hr

(Relative to Wind)

+

Wind Velocity

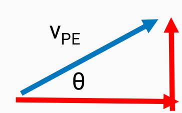
25 km/hr

(Relative to Earth)

v_{WE}

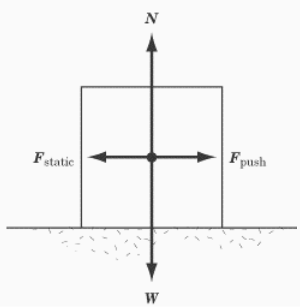
=

Resultant Velocity

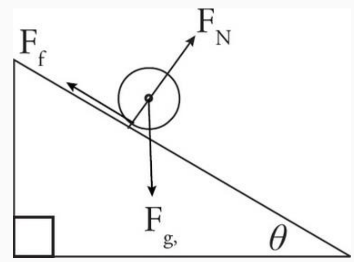


Free Body Diagrams / Force Diagrams

- In a FBD, we represent the object's center of mass as a dot, and show **all** the forces acting on that object as arrows pointing away from the CoM.



- Force diagrams are often used with rotation and torques - they show forces where they act on an object.



Read directions carefully - they should specify which type of diagram they want.

Types of Forces

Field Force: A non-contact force that acts on an object over some distance. For example:

- Gravity (force of attraction between two objects due to their mass)
- Electromagnetism
- Weak Force
- Strong Force

Contact Forces: A force that acts on an object due to direct contact with another object. For example:

- Friction
- Normal Force
- Tension
- Spring Force

On a microscopic level, contact forces are typically caused by electromagnetism between atoms/molecules.

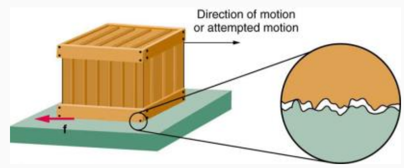
Types of Forces

Normal Force:

- The component of a contact force that acts perpendicular (normal) to the surfaces that are in contact
- Measures how firmly the objects are in contact with each other

Friction:

- The component of a contact force that opposes relative motion between two objects
- The amount of frictional force depends on the normal force and the microscopic properties of the objects



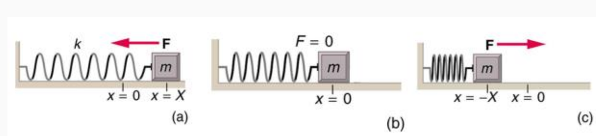
Types of Forces

Tension:

- The force of a rope/string/etc. attached to an object and pulling on it.
- The tension in a rope pulls on the object at both ends of the rope, in the direction of the rope.

Spring Force:

- Similar to tension, but a spring can push or pull an object depending on how it is compressed or stretched
- We'll see spring forces more often as we get into work and energy



What are we doing? (And why are we doing it?)

Newton's 2nd Law of Motion:

$\sum \mathbf{F} = m\mathbf{a}$

Like any of our other vector equations, we can split this into equations for x- and y-components:

$\sum F_x = ma_x$

$\sum F_y = ma_y$

In the Lab:

We had two forces in two dimensions (plus acceleration):

We can choose a convenient coordinate system for solving the problem:

Sometimes, that'll be a "normal" x-y plane, but sometimes it works better to tilt it - forces and accelerations determine what works best

Newton's First Law (Law of Inertia)

"An object in motion will remain in motion and an object at rest will remain at rest unless acted upon by an unbalanced outside force"

- An object moves with a velocity that is constant in magnitude and direction unless a non-zero net force acts on it.
    - If an object is not moving, an unbalanced force is required to make it move
    - If an object is moving, it will continue with a constant velocity unless an unbalanced force causes a **change**

Inertia and Mass

Inertia: the tendency of an object to continue in its original state of motion

(Inertial) Mass: A measure of the object's resistance to change in its motion; objects with greater mass have greater inertia. The greater the mass of an object, the less it is accelerated by a given force.

(We can also define "Gravitational mass" based on the force with which gravity acts on an object. Experimentally, gravitational mass = inertial mass, so we usually just call either one "mass")

Newton's Second Law (F = ma)

The acceleration of an object is directly proportional to the net force acting on it and inversely proportional to its mass.

$$\mathbf{a} = \sum \mathbf{F}/m$$

We usually rewrite this as:

$$\sum \mathbf{F} = m\mathbf{a}$$

Newton's Third Law (Action and Reaction)

"If object 1 and object 2 interact, the force $\mathbf{F_{12}}$ exerted by object 1 on object 2 is equal in magnitude but opposite in direction to the force $\mathbf{F_{21}}$ exerted by object 2 on object 1."

$$\mathbf{F_{12} = - F_{21}}$$

Forces in nature always exist in pairs, as an interaction between two objects.

We represent this in a system schema with arrows that point both directions.

Types of Friction

Kinetic Friction:

- Kinetic friction describes the contact force resisting the relative motion between two objects when one or both are moving
- The direction of the force of friction is always parallel to the plane of contact between the objects (perpendicular to the normal force) and opposite of the direction of motion

Static Friction:

- Static friction describes the contact force resisting the relative motion between two objects when neither object is moving (relative to the other)
- The force of static friction can vary depending on the force it's opposing, up to a maximum amount

Force of Friction

Calculating Frictional Force:

- Both static and kinetic friction depend on two factors:
  - The normal force (which measures how strongly the two objects are pushed together)
  - The coefficient of friction (which measures the microscopic "roughness" of the surfaces that are in contact with each other)
- The coefficient of friction is a property of the two materials that are in contact with each other and has a different value for static and kinetic friction. It's represented by the Greek letter μ.

Coefficient of static friction: μ_s Coefficient of kinetic friction: μ_k

Force of Friction

Calculating Frictional Force:

For either type of friction, the force of friction is given by:

$|\mathbf{F_f}| \leq \mu|\mathbf{F_N}|$ (AP) or $|\mathbf{F_f}| \leq \mu|\mathbf{n}|$ (in your textbook)

We use the μ that corresponds to the type of friction that we're dealing with. The "≤" is due to static friction; the force of kinetic friction is always:

$$F_{fk} = \mu_k F_N$$

The maximum force of static friction is given by:

$$F_{fs.max} = \mu_s F_N$$

Unit 2 Big Idea

If you think a problem can be solved using forces:

1) Draw a diagram (usually a FBD unless the problem specifies otherwise)
2) Choose a logical coordinate system based on your diagram
3) Determine if the forces are balanced (**a** = 0) or not balanced (**a** ≠ 0)
4) Write equations for Newton's 2nd law (in both x- and y-directions if your FBD requires it)

Paragraph Argument FRQs

A paragraph-length response to a question should consist of a coherent argument that uses the information presented in the question and proceeds in a logical, expository fashion to arrive at a conclusion.

Your paragraph should:

- Be clear and to the point
- Be primarily words, though it can include equations or diagrams
- Flow logically from one idea to the next
- Contain a claim, followed by evidence that supports that claim
- Contain relevant physics concepts, explained in words - show that you understand what the equations mean

Paragraph Argument FRQs

Your paragraph should **not**:

- Be mostly equations or diagrams; some are ok, but they're looking for a ***paragraph*** - i.e. words in sentences in an intentional order
- Be a disconnected list of information about the topic - you will get points for the logical connections you make in your writing
- Jump from one idea to another - take time to plan so it flows as a paragraph
- Contain claims without any supporting evidence - be sure to use physics concepts to support all your main ideas

Universal Gravitation

Newton's Law of Universal Gravitation

$$\left|\vec{F}_g\right| = G\frac{m_1 m_2}{r^2}$$

Universal gravitational constant, $G = 6.67 \times 10^{-11}\ \text{m}^3/\text{kg}\cdot\text{s}^2$

Uniform Circular Motion

A moon or planet in orbit is an example of **Uniform Circular Motion.**

It's called "uniform" because the object moves in a circle at a constant speed - it doesn't speed up or slow down, just changes direction

What needs to be true about the velocity and acceleration vectors for this to happen?

Uniform circular motion requires an acceleration that's always perpendicular to the velocity.

Since the direction of velocity is always changing as an object moves in a circle, the direction of acceleration must always change as well.

Where must the acceleration vector always point?

Uniform Circular Motion

What determines the amount of acceleration needed to cause uniform circular motion?

Speed: If two objects move in identical circles, one fast and one slow, which one must accelerate more? (Remember, acceleration is the rate of change of velocity)

Radius: If two objects move in circles at the same speed, one with a large radius and one with a small radius, which one must accelerate more?

Combining these ideas, centripetal (toward the center) acceleration is:

$$a_c = v^2/r$$

Why v^2? See Google Classroom for a video derivation of this equation.

Forces causing circular motion (centripetal force)

Newton's 2nd Law tells us that all accelerations must be caused by a net force

If an object is in uniform circular motion, it has a centripetal acceleration, which must be cause by a net force.

In circular motion, we call this net force "centripetal force". Using Newton's 2nd Law:

$$F_c = ma_c = mv^2/r$$

It's important to remember that "Centripetal force" isn't a new force - it's a title that we give to whatever creates the net force that causes circular motion.

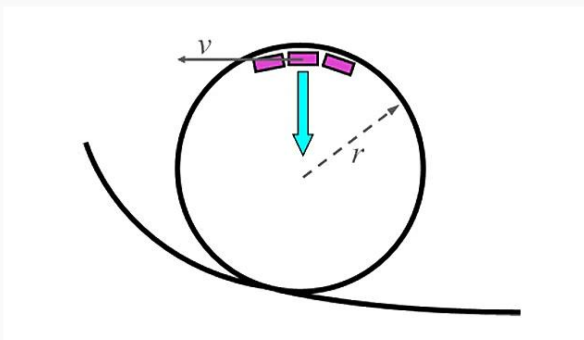
For instance, orbits are caused by gravity - the centripetal force on the moon is caused by the force of gravity between the moon and the Earth

Vertical Circular Motion

Circular motion is often caused by a combination of forces - motion in a vertical circle is a good example.

Vertical circular motion is not necessarily uniform - the speed of the object can change.

Non-uniform circular motion still must have a centripetal acceleration: $a_c = v^2/r$

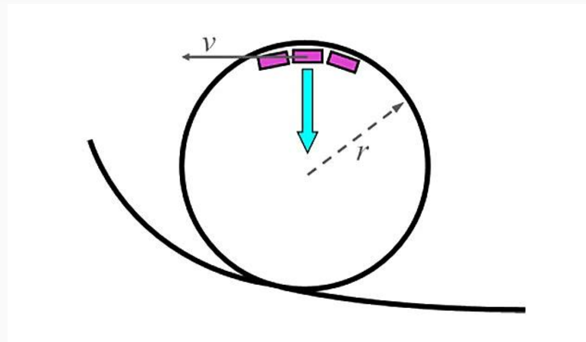


Non-uniform circular motion may also have tangential acceleration - a component of **a** parallel to the velocity (tangent to the circle)

Vertical Circular Motion

When solving problems, it's usually the centripetal acceleration/centripetal force we care about - the component of the net force that acts toward the center of the circle.

The tangential force/acceleration (parallel to the velocity) determines how the object changes speed. It changes, so we don't often use it in calculations.



For equations, it may help to write:

$$\sum F_c = ma_c$$

And, if we care about it: $\sum F_{tan}$ =

Centripetal vs. centrifugal

“Centripetal” means “toward the center” - centripetal force and centripetal acceleration are a necessary part of circular motion

“Centrifugal” means “away from the center” - while there can be centrifugal forces (for example, an upward normal force at the Earth's surface), they do not always occur as part of circular motion

Most things people see or experience as “centrifugal force” are inertia - and a centripetal force that isn't big enough to create uniform circular motion

For example, when turning in your car you slide on your seat toward the outside of the curve - there's no force toward the outside - there just isn't enough frictional force to give you the same centripetal acceleration that the car has

Unit 3 Big Idea

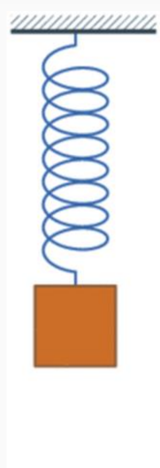
If you think a problem can be solved using forces:

1) Draw a diagram (usually a FBD unless the problem specifies otherwise)
2) Choose a logical coordinate system based on your diagram - usually, this will be centripetal/centrifugal (toward/away from the center) and tangential
3) If something is moving in a circle, forces must be unbalanced - there must be a net force causing acceleration toward the center
4) Write equations for Newton's 2nd law - determine how the forces toward/away from center create the centripetal acceleration

Definitions

Simple Harmonic Motion: Periodic, repeating motion caused by a net restoring force proportional to the displacement from equilibrium position. For example, a mass bouncing on a spring or swinging on a pendulum

Amplitude (A): The maximum distance of an object from its equilibrium position. An object in SHM oscillates between A and -A

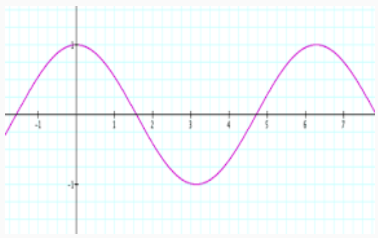


Period (T): The time it takes an object in SHM to complete one full cycle of its motion (e.g. from A to -A and back to A again)

Frequency (f): The number of complete cycles or vibrations per unit of time; the reciprocal of the period (f = 1/T). Typically measured in Hertz (Hz)

Representations of SHM

A graph of position vs time for an object in SHM will be sinusoidal (i.e. a sine or cosine graph):



Based on our definitions of period and amplitude, we can determine an equation for position as a function of time:

Since the oscillator must complete one full cycle per period T, and since the maximum displacement from equilibrium is A (or -A), we get:

$$x = A\cos(2\pi t/T) = A\cos(2\pi f t)$$

We can use this to also determine functions for velocity and acceleration vs. time:

$$v = -2\pi f A\sin(2\pi f t)$$

$$a = -4\pi^2 f^2 \cos(2\pi f t)$$

Note: These equations are all written for radians, not degrees

Types of SHM

The equations on the last slide apply for any type of simple harmonic oscillator; however, the type of oscillator and its properties determine the period of its motion:

Mass on a spring: For a mass m on a spring with spring constant k, the period of oscillation will be:

$$T_s = 2\pi\sqrt{m/k}$$

Pendulum: Though a pendulum is not a perfect example of SHM, it is a good approximation for small angles (roughly 15° or less). Under this approximation:

$$T_p = 2\pi\sqrt{l/g}$$

Tunnel through the Earth: A mass dropped into a tunnel through the center of the Earth would also undergo SHM. Probably not on the AP test, but kinda cool.

Unit 6 Big Ideas

1) **A mass on a spring or pendulum on a string will oscillate with a specific period**
   a) The period depends on the properties of the mass/spring/pendulum
2) These systems will conserve energy - converting back and forth between kinetic and potential; if no external work is done on the system, no energy will be lost, so the oscillations will keep the same amplitude

Experimental Design FRQs

Always 1 experimental design question on the AP Physics 1 test

These questions often involve:

- Designing an experiment using "standard physics lab equipment" - basically anything we've used or similar materials - motion detectors, meter sticks, stopwatches, protractors, carts and tracks, etc.
- Choosing variables to test as part of the experiment and giving them a symbol
- Interpreting hypothetical results (e.g. "If Alex and Brian did an experiment and got these resulting data, what kind of relationship would that show between X and Y?")
- Comparing different models or choosing which model fits data

Experimental Design FRQs

These are often the longer (~25 minute) FRQ questions

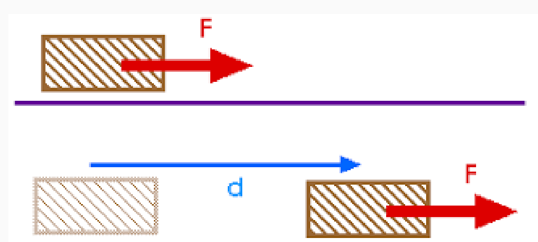
- Most involve steps in addition to designing an experiment (brief short answer questions, graphing, analyzing data, etc.)
- Like other question types, verbs are important:
  - **Compare**: Provide a description or explanation of similarities and/or differences.
  - **Derive**: Perform a series of mathematical steps using equations or laws to arrive at a final answer.
  - **Label**: Provide labels indicating unit, scale, and/or components in a diagram, graph, model, or representation.
  - **Plot**: Draw data points in a graph using a given scale or indicating the scale and units, demonstrating consistency between different types of representations.

Work

In physics, **Work** is done if an object is moved through a displacement while a force is applied to it.

(The force often causes the displacement, but that isn't necessary for work)

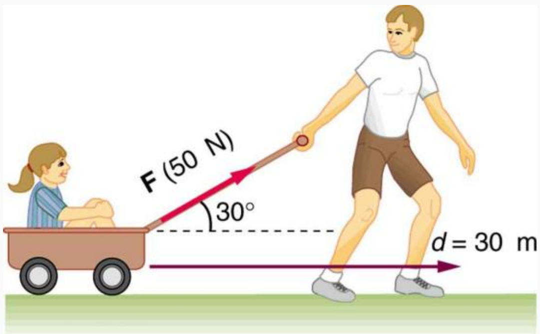
Work only happens when the force (or a component of it) is parallel to the displacement



When force and displacement are in the same direction, a simple work equation is $W = Fd$

Work is measured in Newton-meters, also known as Joules.

Work



When **F** and **Δd** are not parallel, we can use components to calculate the work done.

For this wagon:

In general, we can calculate work using:

$$W = (F\cos\theta)d$$

Where θ is the angle between **F** and **Δd.**

Work and Energy

Last week, we defined **work** in terms of how it's calculated:

$$W = (F\cos\theta)d$$

Another way of defining work is in terms of what it does: work done on an object changes the energy of that object. This is called the **Work-Energy Theorem:**

$$W = \Delta E$$

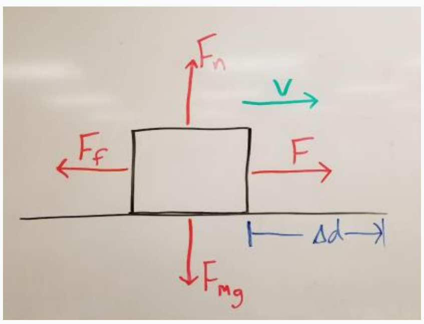
Recall that work is measured in Newton-Meters, also known as Joules.

Energy can also be measured in Joules.

The work-energy theorem tells us that the amount of work done to an object is equal to the change in energy of that object - if we do 10 J of work on something, we increase its energy by 10 J.

More about Work

As mentioned yesterday, work can be either positive or negative - for instance, friction often does negative work on objects



We can also get negative work due to components in opposite directions - for example, carrying an object down a flight of stairs

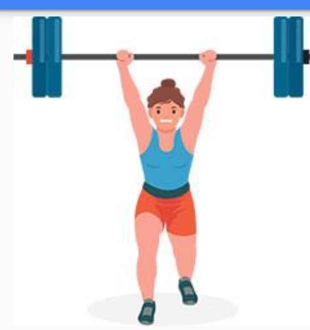
More about Work

It can be important to distinguish between work done *on* something vs. work done *by* something.

If I do positive work *on* an object, that object gains mechanical energy (K, U, or a combination of both).

If positive work is done *by* an object, that object loses energy.

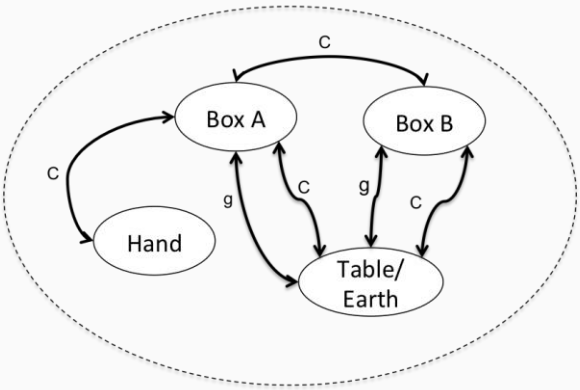
It's important to be be clear about what does the work and what has work done to it. (consider 3rd Law action/reaction pairs of forces)



If you lift a weight, you do positive work on the weight, so it gains potential energy; it does negative work on you, so you lose energy (stored PE in your muscles)

Work and Systems

Like with forces, when calculating work/energy, we can choose a "system" of objects to focus on



Work done by external forces changes the total mechanical energy (K + U) of the system.

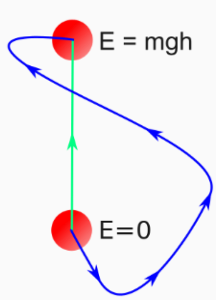
Work done by internal forces transforms energy types within a system.

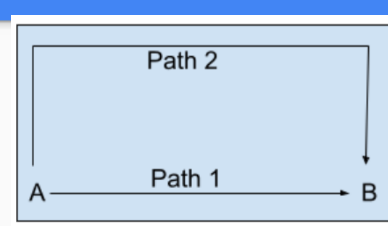
U_g can only exist in a system that includes the Earth; if Earth isn't part of the system, we'd consider how the force of gravity changes the energy of our system.

Conservative and Non-Conservative Forces

A force is conservative if the work it does moving an object between two points is the same no matter the path

Gravity is a common example of a conservative force



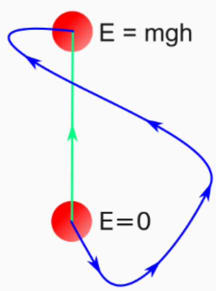


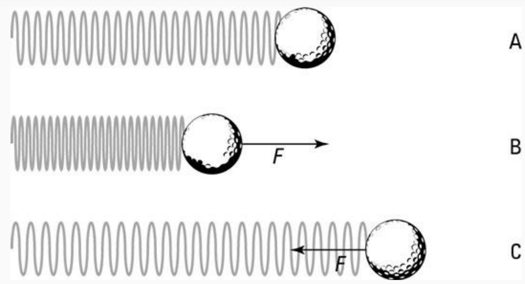
A non-conservative force doesn't have this property - it generally dissipates energy, so energy cannot be regained by reversing the process.

Friction is a common non-conservative force

Conservative Forces and Potential Energy

Conservative forces are the sources of various kinds of potential energy - for instance gravitational potential energy is possible because gravity is conservative

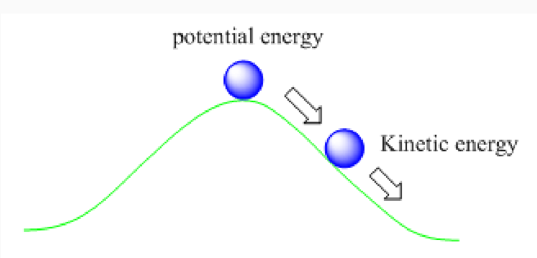




Similarly, spring (or elastic) potential energy exists because spring forces (as we'll see after break) are also conservative forces.

Conservation of Energy

Energy conservation is an idea that applies to all sciences: Energy cannot be created or destroyed, only transferred from one form into another.



In physics, we often describe the law of conservation of energy by saying "The total energy of a closed, isolated system must remain constant"

If we choose a system that doesn't interact with anything outside the system, its total energy must remain constant:

$$E_{initial} = E_{final}$$

Conservation of Energy and Work

Realistically, it's difficult to find a completely isolated system

To accurately express how energy is conserved, we need to account for the work done on the system:

$$E_{initial} + W_{external} = E_{final}$$

We often expand this to include kinetic and potential energy:

$$(K_i + U_i) + W_{ext} = (K_f + U_f)$$

In essence, this equation says: The energy of a system is conserved, unless an external force does work on the system, causing it to change its energy

If no external forces do work on the system, we get the more traditional expression of energy conservation:

$$(K_i + U_i) = (K_f + U_f)$$

Power

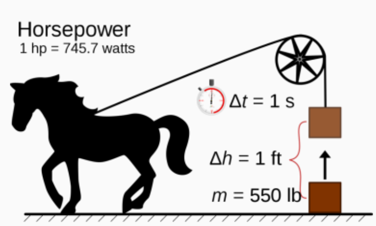
Power, measured in Watts, is defined as the rate at which energy is transferred:

$$P = \Delta E/\Delta t$$

Because of the relationship between work and energy, we can also define power in terms of work done:

$$P = W/\Delta t$$

For example, if it takes you 20 seconds to do 350 J of work, you'd develop 17.5 W of power (Also, you don't "use power" or "provide power" - you almost always "develop power". I'm not sure why.)



Unit 4 Big Ideas

If you think a problem can be solved using energy:

1) Choose a system and decide if energy is conserved within that system. If so:
- Determine the initial and final type(s) of energy; remember that something may have more than one type
- Create an equation for total initial energy = total final energy
1) If energy is not conserved:
- Total initial energy + work done on the system = total final energy
- Identify what external force is doing work, if it's positive or negative work, and how final energy should compare to initial energy

Qualitative/Quantitative Translation

These problems all involve expressing physics concepts in multiple representations - usually, it'll ask you for qualitative reasoning (e.g. "without calculating, will a steeper angle increase or decrease the final height of the ball?"), followed by quantitative reasoning (e.g. "Now, derive an expression for the final height of the ball")

Finally, these questions ask you to connect the two representations (e.g. "Explain why this equation shows that the final height of the ball increases as the angle gets steeper")

If it helps, you can do the question out of order - start with the part you're most comfortable with.

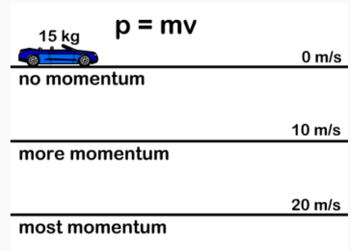
Momentum

In physics, (linear) **Momentum** (**p**) is defined as the product of an object's mass and its velocity:

p = m**v**

Momentum is typically measured in kilogram meters per second (kg · m/s)

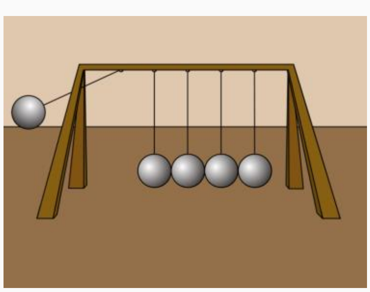
Momentum is a vector quantity that has the same direction as the velocity vector of an object.



Total momentum of a system can be found by adding the (vector) momentum of each object within that system

Conservation of Momentum

When a collision occurs in an isolated system, the total momentum of the system is conserved - both in magnitude and direction.



"Isolated system" means there's no external net force acting on the system - for instance, with the carts and tracks, momentum is conserved as long as friction is negligible.

Like with energy conservation, total momentum is conserved in a collision, but momentum can be transferred from one object to another within the system.

Momentum and Center of Mass

Another way to think about momentum conservation is that the momentum of the *center of mass* of an isolated system must remain constant - this is really just Newton's second law applied to a system: If there is no net external force acting on the system, it must not accelerate, so its velocity must be constant.

Simulation

If the C.O.M. of the system is stationary before a collision, it will remain stationary after the collision.

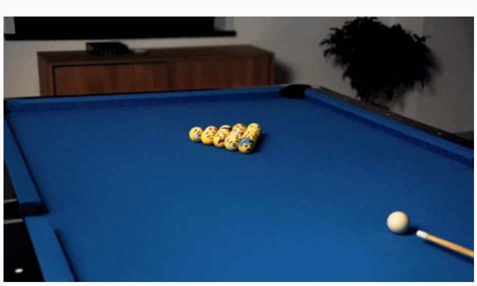
If the C.O.M. of the system is moving before the collision, it will continue moving with the same velocity after the collision.

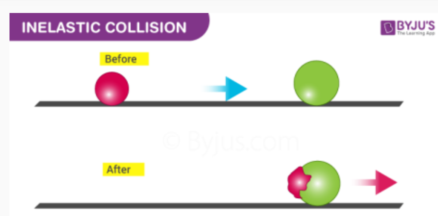
However, unbalanced external forces can change the total momentum of a system

Types of collisions

Collisions can be classified as either **elastic** or **inelastic**.

In an **elastic** collision, both momentum and kinetic energy of the system are conserved





In an **inelastic** collision, momentum is conserved, but kinetic energy is not; a collision where the two objects stick together and move as one is considered *perfectly inelastic*

Forces and impulse

We said previously that momentum is conserved only when a system is isolated - when no net external forces act on the system.

If the system is not isolated, the net force that acts on the system changes its momentum - it causes the mass in the system to accelerate

A force, *F*, delivered over some time interval *Δt*, provides an **impulse**:

$$\mathbf{I} = \mathbf{F}\Delta t$$

The impulse delivered by a force causes the change in momentum of the system it acts on.

Impulse and Momentum

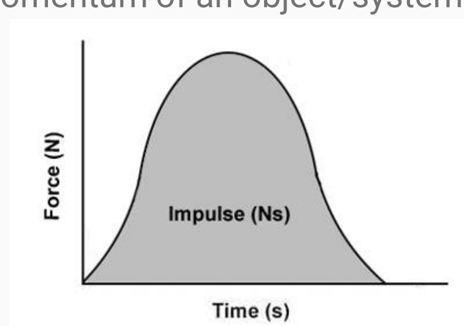
Forces cause a change momentum due to Newton's 2nd Law:

$$F = ma = m(\Delta v/\Delta t)$$

Through some algebra, we can show that the impulse, *I*, that acts on a system is equal to the change in momentum of that system:

$$\mathbf{I} = \mathbf{F}\Delta t = \mathbf{\Delta p} = m\mathbf{\Delta v}$$

Graphically, we can calculate impulse as the area under a graph of Force vs. Time - this area must equal the change in momentum of an object/system.



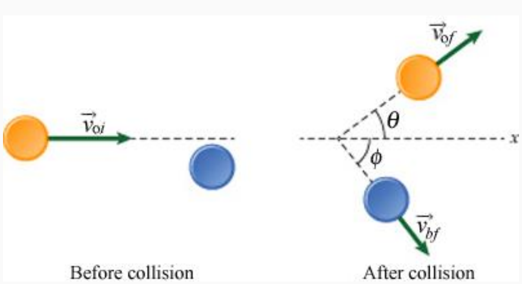
Solving Momentum Problems

When solving momentum problems, consider:

- Is momentum conserved, or is there an external net force that creates an impulse and changes the momentum?
- What type of collision occurs (elastic, inelastic, perfectly inelastic)?
  - If elastic, kinetic energy must be conserved
  - If perfectly inelastic, both objects have the same final velocity

Glancing Collisions / 2-D Momentum

So far, we've examined head-on collisions - both objects moving along a line before and after collisions. Not all collisions behave that way; for example, consider two billiard balls like below:

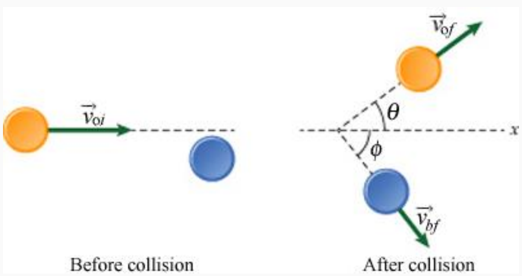


In such a collision, conservation of momentum still applies: the total momentum vector for the system before the collision must equal the total momentum vector after the collision.

Like most other types of problems involving vectors in 2-dimensions, we typically solve these problems by breaking the vectors into components.

Glancing Collisions / 2-D Momentum

First, choose an X- and Y-axis coordinate system (a good choice is probably aligned with one of the vectors, often the initial if only one object is moving)



Then, write a conservation of momentum equation for each component of the momentum:

$$p_{1xi} + p_{2xi} = p_{1xf} + p_{2xf}$$

$$p_{1yi} + p_{2yi} = p_{1yf} + p_{2yf}$$

If we know enough information, this should give us a set of equations that we can solve for our unknown information.

Unit 5 Big Ideas

Most problems will be either solved using forces and kinematics or solved using conservation laws.

If you're solving with conservation laws, consider:

- Is momentum conserved, or is there a net force causing an impulse?
- Is energy conserved in the collision? What type of collision is it?

Then, write equations for conservation of momentum (if it applies) and conservation of energy (if it applies).

Rotational motion definitions

Angular position (θ): The angle at which some object has been rotated relative to some reference position (equivalent to x in translational motion). Traditionally measured in radians.

Angular displacement ($\Delta\theta$): The difference between an object's initial and final angular positions; $\theta_f - \theta_i$; how far the object has been rotated (equivalent to Δx in translational motion). Measured in radians.

Average angular velocity(ω_{av}): The rate at which an object rotates over a given time period. $\omega_{av} = \Delta\theta/\Delta t$. (Equivalent to $\mathbf{v}_{av}$ in translational motion). Measured in rads/s.

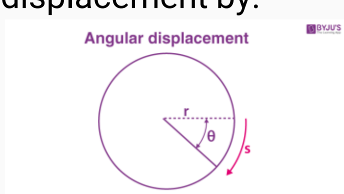
Average angular acceleration (α_{av}): The rate of change of angular velocity over time. $\alpha_{av} = \Delta\omega/\Delta t$. (Equivalent to $\mathbf{a}_{av}$ in translational motion). Measured in rads/s^2.

Instantaneous...: Take the limit as $\Delta t \rightarrow 0$ of either of the above.

Rotational motion relationships

Angular displacement ($\Delta\theta$): When an object rotates, each point on that object travels through an arc. The length of that arc (Δs) is related to the angular displacement by:

$\Delta\theta = \Delta s/r$



Where r is the radius of that arc. We can consider Δs the distance that that point on the rotating object travels

Angular velocity(ω) and angular acceleration (α): Angular velocity and acceleration relate to linear velocity and acceleration in a similar way:

$\omega = v/r$

$\alpha = a/r$

Angular kinematics: Our kinematic equations from unit 1 have equivalent rotational versions:

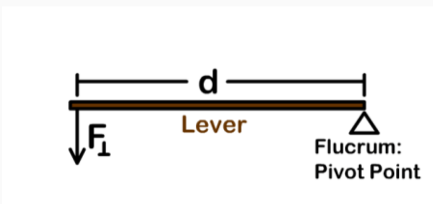
$\theta = \theta_0 + \omega_0 t + 1/2\alpha t^2$

$\omega = \omega_0 + \alpha t$

Forces Causing Rotation

We know from Unit 2 that a net force acting on an object will cause that object (specifically its center of mass) to accelerate.

Depending on how/where that force is directed, it can also cause that object to rotate:



Consider our meter stick and masses from yesterday: Each hanging mass applied a gravitational force to the meter stick; if that force was not at the fulcrum, it caused a rotation about the fulcrum. A force causing a rotation provides a torque.

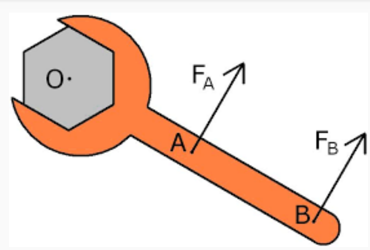
Torque (τ) is the rotational equivalent of force; forces cause linear acceleration, while torques cause angular acceleration

Torque

A force (or component) directed tangential to a pivot point (fulcrum, hinge, etc.) causes a torque. How much torque is given by:

$$\tau = rF\sin\theta$$

Where r is the distance from the pivot point to where the force is applied, F is the amount of force, and θ is the angle between r and F.



Torque is a vector - its direction is along the chosen axis of rotation, and can be determined by the right-hand rule:

$$(\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F})$$

Torque Right-Hand Rule

1. Use your **right** hand (see name of rule)

1. Point fingers in the direction of **r** (from pivot point toward **F**)

1. Curl fingers in the direction **F** points

1. The direction your thumb points is the direction of $\boldsymbol{\tau}$

Equilibrium

In order to be in **mechanical equilibrium**, and object/system must satisfy two conditions:

1. The net force on the object/system must be zero

1. The net torque on the object/system must be zero

An object in equilibrium must have a constant translational velocity (which may or may not be zero) and a constant angular velocity (which may or may not be zero). Stationary objects are a good example of objects in equilibrium.

Torque

As we mentioned yesterday, just like how a net force can cause a translational acceleration, a net torque can cause an angular (rotational) acceleration.

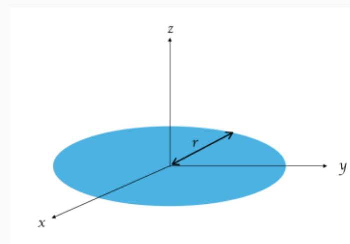
Newton's 2nd Law tells us how net force and acceleration are related:

$$F_{net} = ma$$

We can create a rotational equivalent, but we need a "rotational mass" to do so.

Consider a single particle on a rotating object. Assume a tangential force is applied causing rotation - in other words a torque. We know from yesterday that the force and torque on that particle are related by:

$$\tau = rF$$



Torque

The net torque on that particle is then:

$$\tau = rF_{net} = mra$$

To relate torque to angular acceleration, we can use the relationship between angular and linear acceleration, $a = r\alpha$. Thus:

$$\tau = mr^2\alpha$$

However, this is just torque for one particle on the object

To find the total torque, we'd need to add up all the torques for all the particles on the object:

$$\sum\tau = \sum mr^2\alpha$$

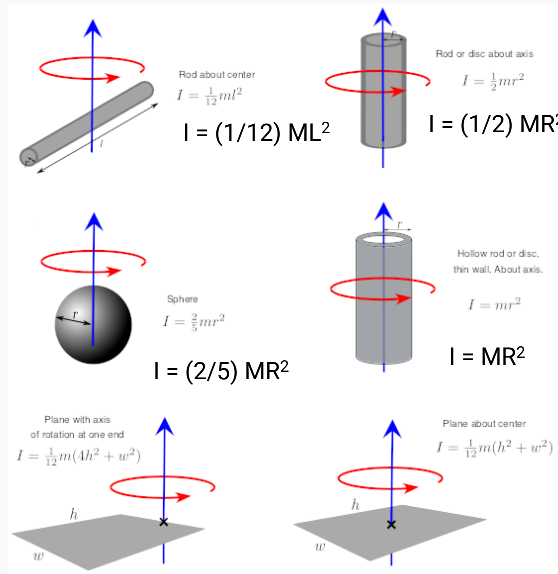
Since the whole object rotates together, α is the same for all particles - m and r are the values that may change. We define the **moment of inertia**, $I = \sum mr^2$. This is our "rotational mass" equivalent. Our 2nd law for rotation is: $\sum\tau = I\alpha$

Moment of Inertia

The moment of inertia, I, is a property of a given object, based on how its mass is distributed relative to its axis of rotation.

Different objects will have different moments of inertia - generally, calculus is needed to determine them, but we can look up common objects.

Like mass, moment of inertia describes the tendency to resist changes in the object's motion.



Rotational Energy

Rotating requires kinetic energy; specifically:

$$K_r = ½I\omega^2$$

Since K_r depends on I, how much energy is used for rotation depends not just on the total mass, but on the distribution of that mass.

For instance, for a mass rolling down a ramp where total energy is conserved:

$$(U_g + K_r + K_t)_{initial} = (U_g + K_r + K_t)_{final}$$

A greater I value means more energy is used for rotation, so less is available for translation.

Angular Momentum

The angular momentum, **L**, of a system is defined as:

$$\mathbf{L} = \mathbf{I}\boldsymbol{\omega}$$

Ideas we learned about linear momentum apply to angular momentum as well. For instance, angular momentum is conserved: If there is no net external torque on a system, then:

$$\mathbf{L_i} = \mathbf{L_f}$$

Consider a gymnast doing a flip. When they push off the ground, they create a torque and give themselves angular momentum. In the air, they tuck - decreasing I and (since L is conserved) increasing ω. To land, they do the reverse, slowing their rotation.

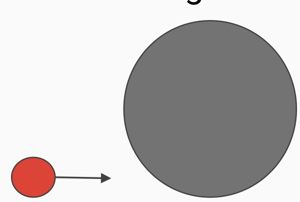
Angular Momentum and Torque

As long as net torque on the system is zero, angular momentum is conserved.

However, if there is an external net torque, it provides an angular impulse (like how an external net force provided a linear impulse). That angular impulse is equal to the change in angular momentum:

$$\tau\Delta t = \Delta L$$

Objects moving linearly can also have angular momentum if they're moving tangential to a pivot point. For instance, consider catching a ball while standing on the turntable:



If the ball is caught near the edge of the turntable, it'd transfer both linear and angular momentum when caught.

Big Ideas to Remember

Circular Motion:
Circular motion requires a centripetal (toward the center) acceleration from a centripetal net force.

Gravitation: There's a law for that. Use the equation.

Energy: Total energy is conserved for closed, isolated systems. Work done on a system changes the energy of the system

Momentum: Momentum is conserved as long as there is no external net force. Remember the difference between elastic/inelastic

Simple Harmonic Motion:
Harmonic motion repeats with a specific period. Makes a sine wave.

Torque and Rotation: There's a rotational version of nearly everything we saw before: kinematics, torque, angular velocity, rotational energy, etc.

Big Ideas to Consider

Conservation Laws:
Is something conserved? Total energy, kinetic energy, linear momentum, angular momentum? If so, write an "initial = final" equation

Static vs. Dynamic:
Is our object stationary? If so, net torque and net force must be zero. If it has angular or linear acceleration, use the 2nd law for torque or for force. Either way, write out the equation and sub in all the forces/torques